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# Explanation of Mean Regression Phenomenon of Capital Market Volatility

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**Abstract:** Mean Reversion is an important phenomenon in financial time series. It means that no matter how a time series rises or falls, it will return to its value center (mean) with high probability. Too much rise will spontaneously fall, too much fall will spontaneously rise. The mean here is not only the arithmetic or geometric mean of time series, but an intrinsic central value. Mean regression theory is one of the predictable theories of financial time series, which poses a challenge to the random walk theory. This article will be based on the US capital market, combined with the three major US stock indexes (Dow Jones index, S&P 500 index, Nasdaq index), from the behavior of market participants, and then explains the mean return phenomenon of capital market volatility.

## 1. Mean Regression Phenomenon of Capital Market Volatility

Firstly, this paper collects the closing prices of the Dow Jones Index, the S&P 500 Index and the Nasdaq Index from 2015 to 2018, calculates the logarithmic return rate, and then calculates the 30-day historical volatility. Each index gets 1007 volatility data. The obtained volatility time series are shown in the following figure.

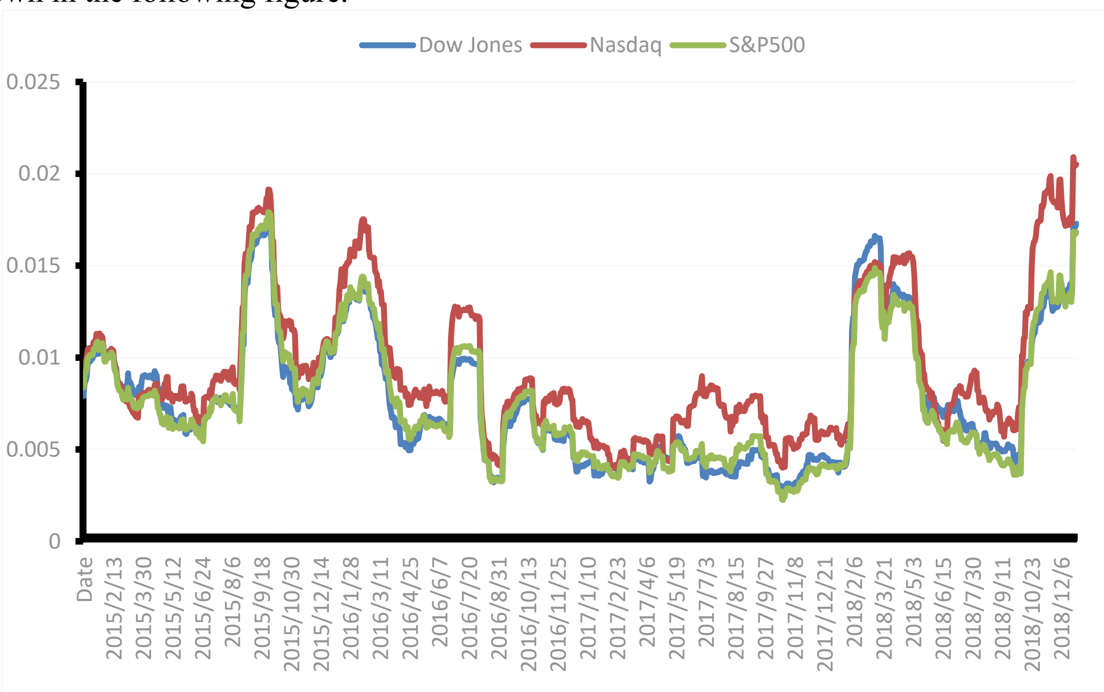


Figure 1 The volatility time series

It is not difficult to find that the volatility of the yields of the three indices has risen sharply after falling sharply or fallen sharply after experiencing a sharp rise. And it seems that there is a long-term average throughout the volatility time series, and the actual volatility fluctuates around the long-term mean.

First, the range of cycle of mean regression which is about to be detected should be limited, because this study is difficult to find a suitable method to test the long-term mean regression characteristics of the volatility time series, since this article will focus on the long-term one. Although financiers have developed numerous methods to detect mean regression characteristics of

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time series, such as ADF test, variance ratio, and Hurst exponent. However, these methods are not applicable when testing volatility mean regression because the mean regression is initially applied in the time series of yields. The rate of return is subject to a random walk, and the future rate of return is unpredictable, but this view has been opposed by some economists. These economists believe that the rate of return time series should follow the mean regression process. If the rate of return rises one day, then the probability of falling next day will be greater, that is, the future rate of return is predictable. However, as can be seen from the volatility sequence diagram in the above figure, the volatility time series has properties that are distinct from the rate of return time series. First, the volatility has a very strong momentum in the short term, that is, the volatility “aggregation” phenomenon-high volatility often follows higher volatility. In other words, the volatility time series reflects a strong tendency in short-term, which does not obey the random walk process. Secondly, the volatility time series will have a trend reversal in medium-term or long-term, that is, a regression phenomenon, but the regression period is longer.

The literature has pointed out that the capital market rate of return volatility time series has the phenomenon of mean regression. For example, Bali & Demirtas (2008) pointed out: The empirical findings indicate that the conditional variance, log-variance, and standard deviation of index futures returns are pulled back to some long-run average level over time. The author has studied the volatility of China's stock market, using ADF test and Hurst index method. We can not get the conclusion of mean regression (the calculated Hurst index is close to 1, which means a strong trend). However, it is found that the autocorrelation coefficient changes from positive to negative at higher order, which provides a basis for mean regression in the medium or long term. Maybe I can draw a satisfactory conclusion by “squeezing” and “interrogating” the data, but I don't like it. In fact, in the volatility of American stock market such phenomenon is also detected that high-order autocorrelation coefficient turns positive to negative, but the order of symbolic reversal is higher, which provides evidence for mean regression in the medium and long term. In a word, the volatility time series has this characteristic: there are two periods t and T, and t < T. The volatility time series has strong trend in period t and mean regression in period T. Below is an autocorrelation sequence of the volatility of the three major stock indices.

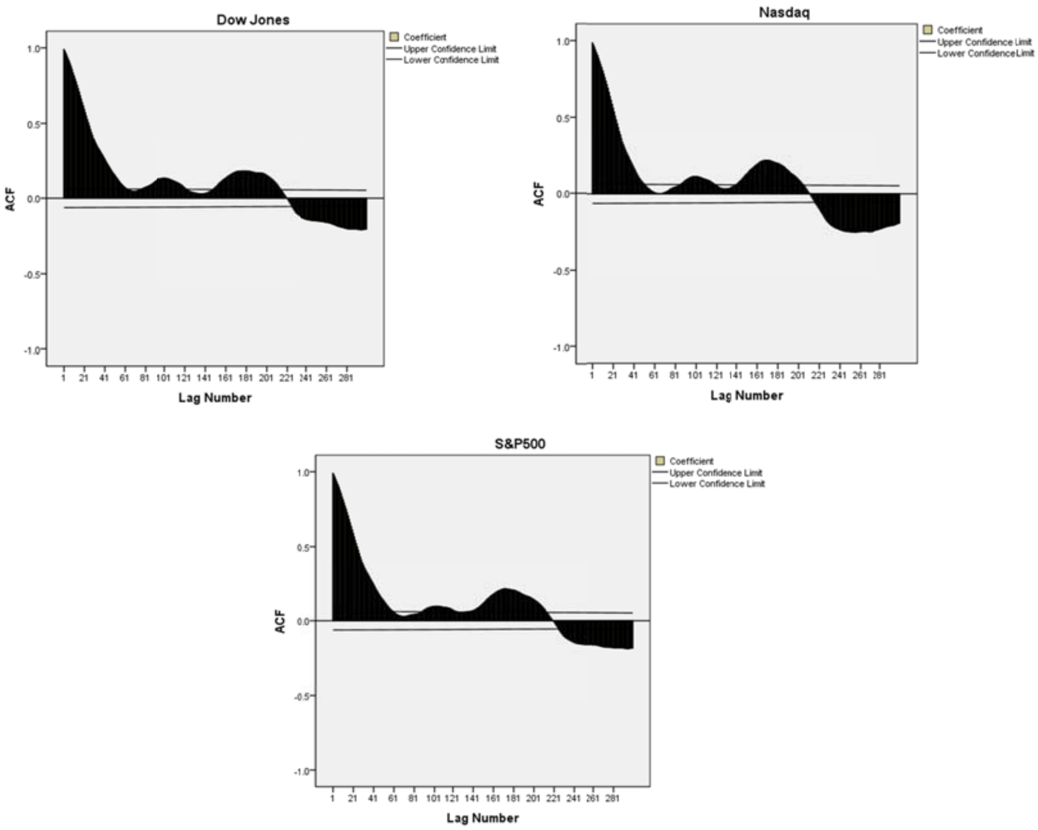


Figure 2 Autocorrelation sequence of the volatility of the three major stock indices

## 2. Explanation of Mean Regression Phenomenon

In recent years, the development of mathematical finance has delivered us a deeper understanding of volatility, but when market abnormalities occur, mathematical models often fail. For example, before the subprime crisis, copula model was widely used in MBS transactions, but after the crisis, copula model became the target of public criticism. Economics is essentially a social discipline that studies human beings. Therefore, as a derivative discipline of economics, finance should also put its thinking on human economic behavior. Next, the article will start from the economic behavior of the people, divide the market participants into long-term investors and short-term speculators, and explain that the mutual game between the two market participants caused the phenomenon of medium- and long-term mean return of volatility.

### 2.1. Long-term investors and short-term speculators together constitute market participants

In modern financial theory, classifying market participants based on different characteristics is a common method. For example, Minsky in the financial instability hypothesis according to whether the cash flow can cover the debt principal and interest divides markets participants into three categories: hedging (complete coverage), speculative (only covering interest), Ponzi (even with interest unable to overwrite).Another example is that, Harrison and Kreps divide investors into “optimistic” investors and “pessimistic” investors when they solve the realistic problem of heterogeneous beliefs.

This paper divides participants in capital market into long-term investors and short-term speculators, both of which are risk averse. Long-term and short-term mean two different holding periods. There are two reasons for such distinction. First, the risk tolerance of long-term investors is low, while that of short-term speculators is high. Second, long-term investors focus on stable capitalization gains, while short-term speculators focus on higher-yield spreads. Therefore, long-term investors hold financial assets for a longer period of time and fewer transactions; short-term speculators trade as much as possible in order to obtain profits from the difference.

There have been many documents pointing out that volume is significantly positively correlated with volatility. Therefore, the more the proportion of short-term speculators, the more transactions in the market, the higher the volatility. The more the proportion of long-term investors, the less transactions in the market, the more the prices tend to remain stable, and the lower the volatility. Therefore, the trading behavior of short-term investors has the power to increase the volatility, and the trading behavior of long-term investors has the power to suppress the volatility. In addition, the literature shows that the fund prefers stocks with high volatility, but the rising proportion of fund holdings has a stabilizing effect on the volatility of the stock. As the main force of long-term investors, funds can prove that long-term investors do have the power to reduce volatility. One assumption to be noted here is that market liquidity must be equivalent enough to make sure that both types of participants will not lose or gain on the liquidity premium.

Assuming that all the long-term investors in a financial asset market are long-term investors, there is a phenomenon of scarce transactions, and most investors hold the asset for a long time. Volatility will be very low, but when the volatility is lower than a certain value, the Sharp ratio of the asset will be at low level and more long-term investors will enter the market for trading, and trading will increase the volatility. Conversely, assuming that the market is full of short-term speculators, there will be excessive trading and high volatility. But when the risk represented by volatility is higher than the risk tolerance of short-term speculators, short-term speculators will choose to exit the market and the transaction will decrease, which in turn will lead to the decrease of excessive volatility. In addition, excessive volatility means that price volatility is fierce, and it is difficult for short-term speculators to profit from stock price fluctuations, causing short-term speculators to withdraw from the market.

### 2.2. Short-term speculators have a crowd-out effect on long-term investors

At present, the theory has shown that the volatility has a “aggregation effect”, that is, higher volatility tends to occur after higher volatility. This article can explain this phenomenon here and as part of the mean regression cycle.

Before considering the problem, the first thing to be clear is that short-term speculators have higher risk tolerance, which means that there is volatility σ1. When the market uncertainty σ>σ1, market fluctuation will be too intense for short-term speculator to continue transaction. Long-term investors have lower risk tolerance, which is the σ2. When the uncertainty σ>σ2, long-term investors will choose to exit the market. Here we must assume σ1>σ2.

The initial market condition is assumed as follows: the market volatility is low, the market runs smoothly, and thereby the future is expectable. At this time, it is difficult for short-term speculators to obtain speculative returns, since market participants' expectations are consistent according to the efficient market theory, according to which future prices are fully reflected in the present. But sooner or later, exogenous shocks will come, causing panic in the market (one of the symbolic is the increase of VIX index, which also means that the expected volatility will rise). In other words, the future is difficult to be expected, and there are divergent expectations of future prices in the market. At this time, speculative funds will flood into the market. As long as short-term speculators correctly predict the future, they can make huge profits. But uncertainty about the future will force some long-term investors to exit the market, and more stocks in the market will be held by short-term speculators.

The continuous influx of such short-term speculators and the continuous extrusion of long-term investors are also the process of rising volatility, which has resulted in 'aggregation effect'. The beginning of a volatility regression cycle should be that volatility rises under the momentum effect.

### 2.3. Excessive volatility causes short-term speculators to exit the market

The previous section discussed the short-term speculators' crowd-out effect on long-term investors, and the volatility continues to rise. We keep on this process. After long-term investors were forced to withdraw from the market, short-term speculators accounted for the participants, and the volatility was higher. What needs to be explained now is that speculators are risk-averse. When the risk in the market is too high, some short-term speculators will also withdraw from the market. At the same time, the market panic cannot be permanently spread, the market will eventually become clear. Short-term speculators can't get excess returns when the market again forms consistent expectations. This is the second process of the regression cycle, when volatility is too high, there will be a downward trend.

### 2.4. Actual volatility fluctuates around equilibrium volatility

There is bound to be an average in the mean regression, but the author tends to define this mean as the equilibrium value. The establishment of equilibrium volatility is based on the category of market participants: long-term investors and short-term speculators. Use L for long-term investors and S for short-term speculators. Long-term investors have a lower volatility limit, $\sigma_L^1$, when the actual operating volatility is lower than this value, long-term investors will get higher utility. Long-term investors also have a volatility cap $\sigma_L^2$, which loses utility when the actual operating volatility is higher than this value. Short-term speculators have upper and lower limits $\sigma_S^1$ and $\sigma_S^2$. Only when the actual operating volatility is higher than the lower limit - when the stock price fluctuates, short-term speculators can obtain speculative gains, and lose utility when they are below the lower limit. When volatility is above the upper limit, short-term speculators also lose utility.

It is assumed here that $\sigma_L^1 < \sigma_L^2$, $\sigma_S^1 < \sigma_S^2$, $\sigma_L^1 < \sigma_S^2$, $\sigma_L^1 < \sigma_S^2$; size relationship between $\sigma_L^2$ and $\sigma_S^1$ is to be determined. Based on this assumption, five volatility intervals were constructed:

1) When $\sigma_S^1 < \sigma_L^2$;


Figure 3 Five volatility intervals 1

2) When $\sigma_S^1 > \sigma_L^2$.

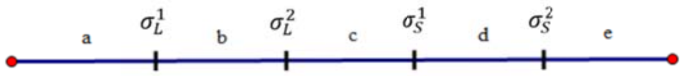


Figure 4 Five volatility intervals 2

The following is a game tree to describe the operating mechanism of volatility. There are four rules for building a game tree:

1) Assuming that the L and S potentials are balanced in the market under the initial state, S first makes a choice and then L makes a choice.

2) S choose 'speculate' or 'not speculate', L choose 'decrease holding', 'wait and see' or 'increase holding'.

3) 'decrease holding' makes volatility enter the next interval, 'wait and see' has no effect on volatility, 'increase holding' makes volatility enter the last interval; 'speculate' makes volatility enter the next interval; 'not speculate' has no effect on volatility. The results of the two choices can be superimposed or offset each other.

4) In the interval a, since the volatility is too low, L only selects 'increase holding' and makes the volatility enter the next interval. In the interval e, since the volatility is too high, S only selects 'not speculate' and moves the volatility into the previous interval.

Let's assume the utility values obtained by L and S:

$$U_L = \begin{cases} 1, \sigma < \sigma_L^1 \\ 1, \sigma_L^1 < \sigma < \sigma_L^2 \\ 0, \sigma_L^2 < \sigma \end{cases} \qquad U_S = \begin{cases} 0, \sigma < \sigma_S^1 \\ 1, \sigma_S^1 < \sigma < \sigma_S^2 \\ 0, \sigma_S^2 < \sigma \end{cases}$$

The assumption here is that utility values are easily misinterpreted. Note that this is the utility of long-term investors and short-term speculators on volatility (or risk).

In the following, the game tree will be constructed, which is divided into $\sigma_S^1 < \sigma_L^2$ and $\sigma_S^1 > \sigma_L^2$. The initial value is in the range of a, b, c, d and e, respectively, and the utility value obtained by L and S.

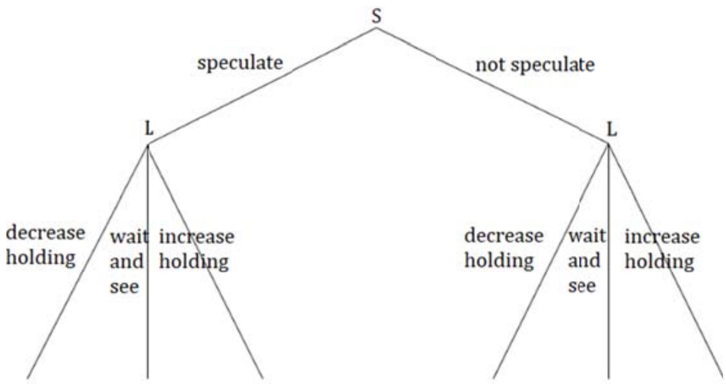


Figure 5 Game tree

In the case of $\sigma_S^1 < \sigma_L^2$, that is, when the volatility boundaries of long-term investors overlap with those of short-term speculators, the utility value (1,1) is always an equilibrium solution regardless of which volatility interval the initial state is in. That is to say, regardless of the range of volatility, long-term investors and short-term speculators will adopt strategies to return volatility to interval c.

In the case of $\sigma_S^1 > \sigma_L^2$, the risk tolerance interval of long-term investors and short-term speculators does not intersect, and only zero-sum games can occur. Regardless of the initial interval, both parties will adopt strategies to maximize their utility and eventually return to the symmetric interval b or interval d centered on the c-interval.

In summary, the expectation of $\sigma_S^1$ and $\sigma_L^2$, i.e. the median value of c interval, is taken as the equilibrium volatility $\sigma_E$.

In case 1, the volatility will return to the c interval centered on the $\sigma_E$; in case 2, the volatility will fluctuate around the $\sigma_E$. Then define the equilibrium volatility $\sigma_E$ as:

$$\sigma_E = \frac{\sigma_L^2 + \sigma_S^1}{2}$$

Among equation above, $\sigma_L^2$ is the largest risk level that long-term investors can tolerate, and $\sigma_S^1$ is the lowest volatility above which short-term speculators can profit. The above proves that there is an equilibrium volatility, and the actual operating volatility should fluctuate around the equilibrium volatility.

Table 1: Values of utility of strategies combinations

| Situation 1: $\sigma_S^1 < \sigma_L^2$ | | | | | | |
|---|---|---|---|---|---|---|
| ($U_L$,$U_S$) strategy / Initial interval | Speculate, Decrease holding | Speculate, Wait and see | Speculate, Increase holding | Not speculate, Decrease hodling | Not speculate, Wait and see | Not speculate, Decrease holding |
| a | 1,1 | 1,0 | 1,0 | | | 1,0 |
| b | 0,1 | 1,1 | 1,0 | 1,1 | 1,0 | 1,0 |
| c | 0,0 | 0,1 | 1,1 | 0,1 | 1,1 | 1,0 |
| d | 0,0 | 0,0 | 0,1 | 0,0 | 0,1 | 1,1 |
| e | | | | 0,0 | 0,1 | 1,1 |
| Situation 2: $\sigma_S^1 > \sigma_L^2$ | | | | | | |
| a | 0,0 | 1,0 | 1,0 | | | 1,0 |
| b | 0,1 | 0,0 | 1,0 | 0,0 | 1,0 | 1,0 |
| c | 0,0 | 0,1 | 0,0 | 0,1 | 0,0 | 1,0 |
| d | 0,0 | 0,0 | 0,1 | 0,0 | 0,1 | 0,0 |
| e | | | | 0,0 | 0,1 | 0,0 |

## 2.5. More Explanations of Game Model

In the analysis of the last section, the game model is constructed in two cases. The first case is $\sigma_S^1 < \sigma_L^2$. In this case, no matter where the initial state volatility is, the next period volatility will enter the c-interval. When the initial state is in the c-interval, the subsequent multi-period volatility should be in the c-interval. Therefore, when $\sigma_S^1 < \sigma_L^2$ is used, the actual operating volatility only fluctuates in the C range. The second case is $\sigma_S^1 > \sigma_L^2$, in which there is no equilibrium solution. If the game in this situation continues to expand into multiple periods, according to the principle of utility maximization, both long-term investors and short-term speculators have incentives to adopt strategies to make volatility enter the range that maximizes their utility. Therefore, the volatility in this case does not remain within an inter-cell fluctuation, but the equilibrium volatility around the c-interval fluctuates with a larger amplitude.

The relative position of the $\sigma_S^1$ and $\sigma_L^2$ determines the fluctuation pattern of the actual operating volatility, so how to determine the size of the $\sigma_S^1$ and $\sigma_L^2$? First of all, $\sigma_S^1$ should be determined by the short-term speculators' own ability. The more capable speculators should have higher $\sigma_S^1$, because they can grasp the more severe fluctuations and profit from them. With the development of quantitative trading, the more speculators are able to grasp the more violent fluctuations, therefore $\sigma_S^1$ should have an upward trend in the future. Furthermore, as for $\sigma_L^2$, the highest level of risk that long-term investors are willing to bear should depend on market conditions. Due to the limitations of short-selling, it is difficult for investors to make profits in a bear market, so optimistic expectations of long-term investors in a bull market will increase their willingness to bear higher risks. On the contrary, $\sigma_L^2$ should be lower in a bear market, because long-term investors do not want to lose utility on risk even when it is difficult to obtain investment returns. In a word, $\sigma_S^1$ and $\sigma_L^2$ should be regarded as an exogenous variable.

## 3. Conclusion

Based on the characteristics of mean regression and the behavior of market participants, this paper briefly describes the specific process of a volatility regression cycle, and then establishes a game model. It discusses the viewpoint that actual volatility should fluctuate around the equilibrium volatility, thus explaining why the volatility of capital market presents the characteristics of mean regression.

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